Solutions to JEE Advanced Home Practice Test -4 |JEE 2024 | Paper-1

PHYSICS

1.(A) Let the exposure time be t and the uncertainty in its value be Δt

$$\omega = \frac{33.3 \times 360°}{60} = 200° s^{-1} \; ; \quad t = \frac{\theta}{\omega} = \frac{10.8}{200} = 0.054 s$$

$$And \left|\frac{\Delta t}{t}\right| = \left|\frac{\Delta\theta}{\theta}\right| + \left|\frac{\Delta\omega}{\omega}\right| \Rightarrow \Delta t = t\left|\frac{\Delta\theta}{\theta}\right| + t\left|\frac{\Delta\omega}{\omega}\right| = \frac{|\Delta\theta|}{\omega} + \frac{|\Delta\omega| t}{\omega} = \frac{0.1}{33.3} + \frac{0.1 \times 0.054}{33.3} = 0.003$$

$t = (0.054 \pm 0.003) s$

2.(D) $h_2 = R\left(2^{\frac{1}{3}} - 1\right)$

Acceleration inside the earth at a distance r form the centre

$$g_1 = \frac{GM}{R^3}.r = \frac{GM}{R^2}\left(\frac{R-h_1}{R}\right) = g\left(1 - \frac{h_1}{R}\right); \qquad \Delta g_1 = \left(\frac{dg_1}{dh_1}\right)\Delta h_1 = -\frac{g}{R}\Delta h_1$$

$\Delta h_1 = +1 km \quad \therefore \quad \Delta g_1 = -\frac{g}{R}(1km) \quad ....(i)$

[This is independent of depth h_1]

At a distance r from the centre outside the earth

$$g_2 = \frac{GM}{r^2} = \frac{GM}{R^2}\frac{R^2}{r^2} = g\frac{R^2}{(R+h_2)^2} \quad \therefore \quad \Delta g_2 = \left(\frac{dg_2}{dh_2}\right)(\Delta h_2) = \frac{-2gR^2}{(R+h_2)^3}\Delta h_2$$

$\Delta h_2 = 1 km$

$$\Delta g_2 = \frac{2gR^2}{(R+h_2)^3}(1km) \quad ....(i)$$

Since $\Delta g_1 = \Delta g_2 \quad \therefore \quad -\frac{g}{R}(1km) = -\frac{2gR^2}{(R+h_2)^3}(1km)$

$$\therefore \quad (R+h_2)^3 = 2R^3 \quad \Rightarrow \quad 1 + \frac{h_2}{R} = (2)^{1/3} \qquad h_2 = R[2^{1/3} - 1]$$

3.(B) $\frac{1}{f_1} = \left(\frac{7}{5} - 1\right)\left(\frac{-1}{50} - \frac{1}{50}\right); \qquad \frac{1}{f_1} = \frac{-2}{5} \times \frac{2}{50}$

$\frac{1}{f_1} = \frac{-2}{125}; \qquad f_1 = -\frac{125}{2} cm$

$\frac{1}{f_2} = \left(\frac{3}{2} - 1\right)\left(\frac{1}{50} + \frac{1}{50}\right); \qquad \frac{1}{f_2} = \frac{1}{2} \times \frac{2}{50} = \frac{1}{50}$

$f_2 = 50\, cm\, ; \quad \frac{1}{v_1} - \frac{1}{u_1} = \frac{1}{f_1}$

$$\frac{1}{v_1}-\frac{1}{(-100)}=-\frac{2}{125};\ \frac{1}{v_1}=-\frac{1}{100}-\frac{2}{125}$$

$$\frac{1}{v_1}=\frac{-5-8}{500};\quad v_1=-\frac{500}{13}cm$$

$$u_2=-\frac{500}{13}-\frac{800}{13};\quad u_2=-\frac{1300}{13}=-100\ cm$$

$$\frac{1}{v_2}-\frac{1}{u_2}=\frac{1}{f_2};\ \frac{1}{v_2}-\frac{1}{(-100)}=+\frac{1}{50};\quad v_2=100$$

4.(B) At time t ; $\frac{dN}{dt}=t^2-\lambda N \quad\Rightarrow\quad \frac{d^2N}{dt^2}=2t-\lambda\frac{dN}{dt} \quad \left(\frac{d^2N}{dt^2}=0\right)$ for minimum

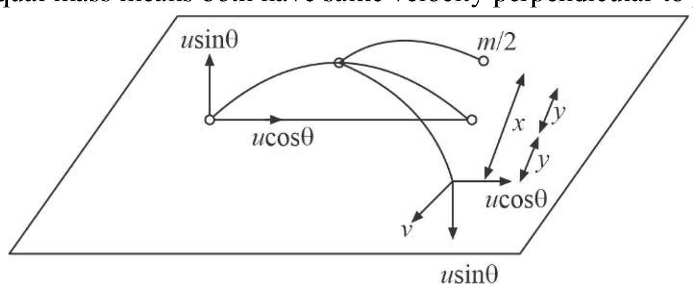
$$\Rightarrow\quad 0=2t_0-\lambda(t_0^2-\lambda N_0)\ \Rightarrow\quad N_0=\frac{\lambda t_0^2-2t_0}{\lambda^2}$$

Solution for Q. 5 – 6

5.(50)

6.(150)

- Body lands after travelling same horizontal distance along projection means horizontal velocity remains same (in plane).
- Fragments lands at the same time means fragments have same vertical velocity. By momentum conservation in vertical, it will be zero.
- They are of equal mass means both have same velocity perpendicular to plane of projection.



- Increase in KE is due to velocity which is perpendicular to plane.

$$\left(\frac{1}{2}mu^2\right)\times\frac{1}{2}=\frac{1}{2}\left(\frac{m}{2}\right)v^2\times 2$$

$$\frac{1}{4}mu^2=\frac{1}{2}mv^2;\qquad \frac{u}{\sqrt{2}}=v$$

$$y=\frac{u}{\sqrt{2}}\times\left(\frac{u}{g}\times\sin 30^\circ\right);\qquad x=2y=\frac{u^2}{\sqrt{2}g}=\frac{100}{\sqrt{2}\times 10}=\sqrt{50}$$

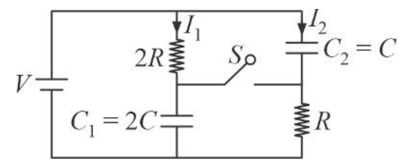
$$V_{\text{Landing}}=\sqrt{u^2+v^2}=\sqrt{u^2+\left(\frac{u}{\sqrt{2}}\right)^2}=\sqrt{\frac{3}{2}u^2}=u\sqrt{\frac{3}{2}}=10\times\sqrt{\frac{3}{2}}=\sqrt{150}$$

Solution for Q. 7 – 8

7.(0.50)

8.(8)

By KVL



$$V - I_2R - \frac{q_2}{C} = 0; \qquad 0 - R\frac{dI_2}{dt} - \frac{dq_2}{Cdt} = 0$$

$$\frac{dI_2}{I_2} = -\frac{dt}{RC} \qquad \text{at } t = 0 \ I_2 = \frac{V}{3R} \ ; \qquad I_2 = \frac{V}{3R}e^{-t/RC}$$

At $\quad t = RC\ln 2$

$$I_2 = \frac{V}{3R}e^{-\frac{RC\ln 2}{RC}}; \qquad I_2 = \frac{1}{2}Amp.; \qquad I_2 = 0.5\ Amp.; \qquad V_1 = \frac{V}{3}$$

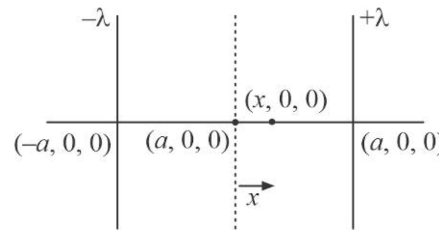
$$q = 2C \times \frac{V}{3} = \frac{2}{3} \times 1 \times 12 = 8\ \mu C$$

Solution for Q. 9 – 10

9.(0.79)

10.(0)

Electric field due to wires



$$\vec{E} = -\frac{2K\lambda}{a-x}\hat{i} - \frac{2K\lambda}{a+x}\hat{i} \ ; \qquad E = -\left(2K\lambda\left(\frac{1}{a-x} + \frac{1}{a+x}\right)\right)$$

$$+\frac{dV}{dx} = +2K\lambda\left(\frac{1}{a-x} + \frac{1}{a+x}\right); \qquad dV = 2K\lambda\left(\frac{dx}{a-x} + \frac{dx}{a+x}\right)$$

$$\Delta V = 2K\lambda(-\ln(a-x) + \ln(a+x))\Big|_0^{a/2}$$

$$\Delta V = 2K\lambda\left[\left(-\ln\left(\frac{a}{2}\right) + \ln a\right) + \left(\ln\left(a + \frac{a}{2}\right) - \ln a\right)\right] = 2K\lambda\left(\ln 2 + \ln\frac{3}{2}\right)$$

$$\Delta V = 2K\lambda\ln 3$$

$$W_1 = Q\Delta V = 2K\lambda Q\ln 3 = 2 \times 9 \times 10^9 \times 40 \times 10^{-6} \times 1 \times 10^{-6} \times 1\cdot 1 = 792 \times 10^{-3}\ \text{J} = 0\cdot 79\ \text{J}$$

$$dW_2 = \overrightarrow{QE} \cdot \overrightarrow{dr} = 0; \qquad W_2 = 0$$

11.(ABCD)

Translational motion

$$F - f = M\ a_{COM}$$

Rotational motion

$$F \times (R + x) = (I_{COM} + MR^2)\alpha; \qquad \alpha = \frac{F(R+x)}{I_{COM} + MR^2}$$

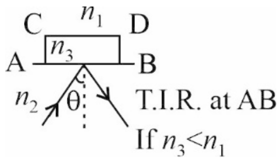
Point of contact

$$a_{COM} = R\alpha \quad ; \qquad a_{COM} = \frac{F(R+x)R}{I_{COM} + MR^2}$$

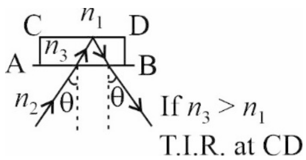
$$f = F - M\left(\frac{F(R+x)R}{I_{COM} + MR^2}\right); \quad f = F\left(\frac{I_{COM} + MR^2 - MR^2 - MRx}{I_{COM} + MR^2}\right); \qquad f = \frac{F(I_{COM} - MRx)}{(I_{COM} + MR^2)}$$

12.(ACD)

(A)



(B)



(C) T.I.R. for both $n_3 < n_1$ *or* $n_3 > n_1$

13.(ABC) Consider a tooth on the front sprocket. It gives this speed, relative to the frame, to the link of the chain it engages:

$$v = r\omega = \left(\frac{0.16m}{2}\right)(75\ rev/\min)\left(\frac{2\pi\ rad}{1 rev}\right)\left(\frac{1\min}{60s}\right) = \frac{\pi}{5} m/s$$

Consider the chain link engaging a tooth on the rear sprocket.

$$\omega = \frac{v}{r} = \frac{\frac{\pi}{5} m/s}{(0.080m)/2} = 5\pi\ rad/s$$

Consider the wheel tread and the road. A thread could be unwinding from the tire with this speed relative to the frame:

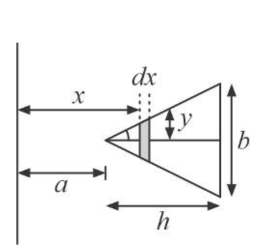
$$v = r\omega = \left(\frac{0.80m}{2}\right)(5\pi\ rad/s) = 2\pi\ m/s$$

14.(AC) $\frac{hc}{\lambda_L} = 13.6\left(\frac{1}{(n-1)^2} - \frac{1}{n^2}\right) \quad \Rightarrow \quad \lambda_L$ is proportional to $\frac{(n(n-1))^2}{(2n-1)}$

$\Delta p_L =$ Momentum of emitted photon $= \frac{h}{\lambda_L} \Rightarrow \quad \Delta p_L$ is proportional to $\frac{(2n-1)}{(n(n-1))^2}$

$\frac{hc}{\lambda_M} = 13.6\left(\frac{1}{(n-1)^2}\right) \quad \Rightarrow \quad \lambda_M$ is proportional to $(n-1)^2$

15.(AD) $d\phi = \left(\frac{\mu_0 i}{2\pi x}\right)\cdot\left(\frac{b}{h}(x-a)dx\right)$; $\quad \int d\phi = \int_a^{a+h} \frac{\mu_0 i}{2\pi}\left[\frac{b}{h}\left[1-\frac{a}{x}\right]\right]dx$



$\phi = \frac{\mu_0 i}{2\pi}\frac{b}{h}\left[x - a\ln x\right]_a^{a+h}$; $\quad \phi = \frac{\mu_0 i}{2\pi}\frac{b}{h}\left[h - a\ln\left(\frac{a+h}{a}\right)\right]$

$\phi = \frac{\mu_0 i}{2\pi}\frac{b}{h}[h - a\ln 2]$; $\quad M = \frac{\phi}{I} \Rightarrow M = \frac{\mu_0}{2\pi}\frac{b}{h}[h - a\ln 2] = \frac{4\pi\times10^{-7}}{2\pi}\times 2\times[1-\ln 2]$

$M = 4\times10^{-8}[1-\ln 2]$

16.(ACD) $p_A = p + p_0 + \rho gh$ ... (i)

Apply Bernoulli is theorem at A and C and E

$$(p + p_0) + \rho gh = p_c + \frac{V_c^2}{2}$$

$$= p_0 + \rho\frac{V_E^2}{2} \qquad \text{... (ii)}$$

$p_c = p_0 + \rho g h_1$; $\quad A_C V_C = A_E V_E$; $\quad A_C V_C = \left(\frac{A_C}{2}\right)V_E$

From equation. (i),

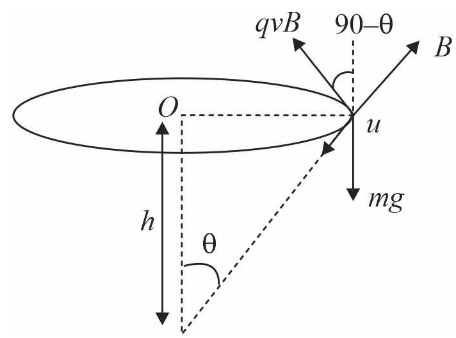
$$(p_0 + \rho g h_1) + \rho\frac{V_c^2}{2} = p_0 + \rho(2V_c)^2; \quad \rho g h_1 = 4\rho V_c^2 - \frac{\rho}{2}V_c^2 = \frac{7}{2}\rho(V_c)^2 \quad \text{or} \quad h_1 = \frac{7V_c^2}{2g}$$

From equation. (ii),

$$(p + p_0) + \rho gh = (p_0 + \rho g h_1) + p\frac{V_c^2}{2}; \; p + \rho gh = \rho\left(\frac{7V_c^2}{2}\right) + \rho\left(\frac{V_c^2}{2}\right); v_c^2 = \frac{(p + \rho gh)}{4\rho}$$

Discharge rate $= A_c V_c = A\left(\frac{(p + \rho gh)}{4\rho}\right)^{1/2}$

17.(1) $qvB \sin\theta = mg$



$$qvB \cos\theta = \frac{mv^2}{h\tan\theta}$$

$$qvB = \frac{mv^2}{h\tan\theta}$$

$$mg = \frac{mv^2}{h}$$

$\Rightarrow n = 1$

18.(5) (given), differentiating w.r.t. time t, we get, $\quad 2x\frac{dx}{dt} = 500\frac{dy}{dt}$

Again differentiating w.r.t. time t, we get, $\quad 2\left[x\frac{d^2x}{dt^2} + \left(\frac{dx}{dt}\right)\left(\frac{dx}{dt}\right)\right] = 500\frac{d^2y}{dt^2}$

Given, $\frac{dx}{dt} = 360 \times \frac{5}{18} ms^{-1} = 100 ms^{-1}$ and is constant.

So, $\frac{d^2x}{dt^2} = 0 \qquad \therefore \qquad 2\left(\frac{dx}{dt}\right)^2 = 500\frac{d^2y}{dt^2}$

$$\Rightarrow \quad \frac{d^2y}{dt^2} = \frac{2(100)^2}{500} = 40 = 4g = ay$$

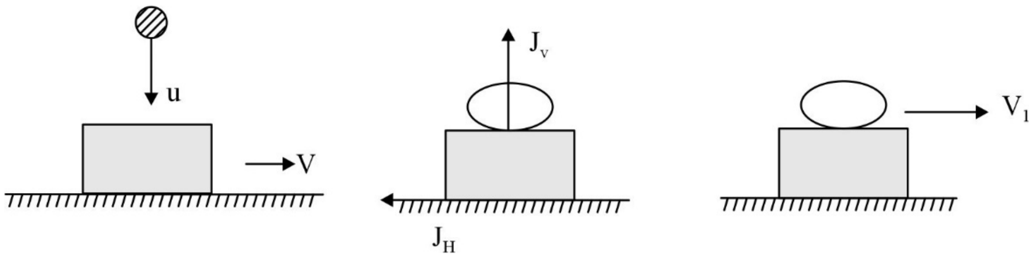
$\therefore$ Effective value of g is $g_{eff} = 4g + g = 5g \qquad \therefore \qquad \beta = 5$

19.(6) During the interaction period of the clay ball and the block, the vertical impulse (due to normal force) applied by ground is

$j_v = \int N\,dt =$ change in momentum in vertical direction $= 1 \times 10 = 10 kgm/s \ldots\ldots\ldots\ldots(i)$

Horizontal impulse of ground friction during the same period is

$j_H = \int \mu\, N\, dt = 0.4 \times 10 = 4 kg\ m/s \quad \ldots\ldots\ldots(ii)$



Velocity (V_1) of the $(block\ +\ ball)$ system just after the impact is given by

$(M+m)V_1 = MV - J_H; \qquad V_1 = \frac{5 \times 2 - 4}{6} = 1\ m/s$

Let the velocity of $(M+m)$ be reduced to V_2 [due to friction] in interval Δt.

At this point another clay ball hits the block.

J_v and J_H given by (i) and (ii) remain same for the second impact

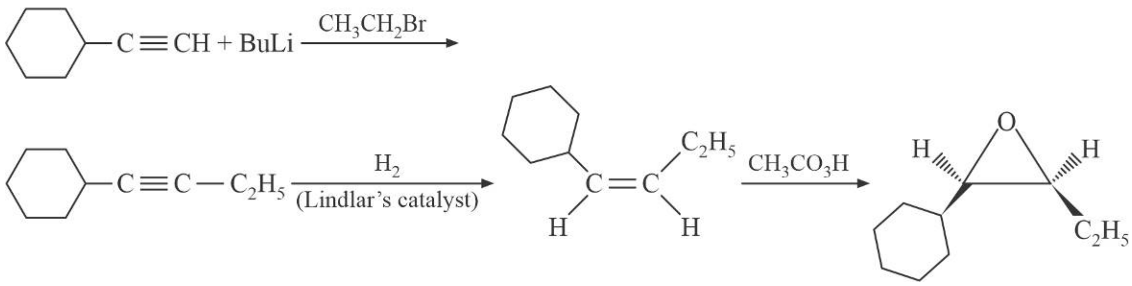
$\therefore \qquad 0 = (M+m)V_2 - J_H \quad \therefore \qquad V_2 = \frac{4}{6} = \frac{2}{3} m/s$

During the interval between two impact, friction causes a retardation of

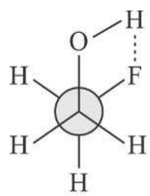
$a = \mu g = 4 m/s^2 \qquad \therefore \qquad V_2 = V_1 - a\Delta t \qquad \Rightarrow \qquad \Delta t = \frac{1 - \frac{2}{3}}{4} = \frac{1}{12}\text{sec}.$

CHEMISTRY

20.(D)



21.(C)



(ii)

Intra-molecular H-bonding stabilizes the conformer.

Transformation of (i) to (ii) will be an exothermic process

22.(C) P.F. of diamond $= \dfrac{n_{eff}.(V_{atom})}{V_{cube}} = \dfrac{8\left[\frac{4}{3}\pi R^3\right]}{a^3} = \dfrac{8\left[\frac{4}{3}\pi R^3\right]}{\left(\frac{8R}{\sqrt{3}}\right)^3} = 0.34$

23.(D) $[Co(H_2O)_6]^{2+}$ has d^7 configuration and sp^3d^2 hybridization.

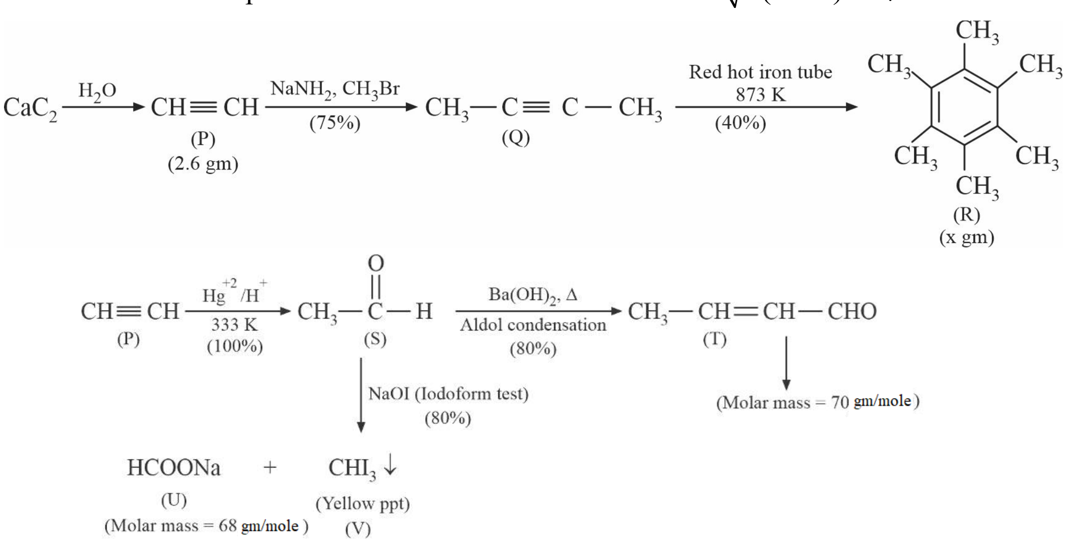
Number of unpaired electron = 3

$\therefore \quad m = \sqrt{n(n+2)} = \sqrt{3\times 5} = 3.87$ BM

$[Fe(NH_3)_6]^{2+}$ has d^6 configuration and sp^3d^2 hybridization.

$\therefore$ Number of unpaired electrons = 4 $\quad\therefore\quad m = \sqrt{4(4+2)} = \sqrt{24} = 4.89$ BM

24.(1.62)



Molar mass of P = 26 gm/mole.

$$P \xrightarrow{75\%} Q$$

(0.1 mole) $\left(0.1 \times \frac{3}{4} = 0.075 \text{ mole}\right)$

$$3Q \xrightarrow{40\%} R$$

(0.075 mole) $\left(\frac{1}{3} \times 0.1 \times \frac{3}{4} \times 0.4 = 0.01 \text{ mol}\right)$

So moles of R = 0.01 mole

Molar mass of R = 162 gm/mole So, x = 0.01 × 162 = 1.62 gm $\Rightarrow$ 1.62 gm

25.(8.24)

Molar mass of U = 68 gm/mole

Molar mass of T = 70 gm/mole

$$P \xrightarrow{(100\%)} S$$

(0.1 mole) (0.1 mole)

$$2S \xrightarrow{(80\%)} T$$

(0.1 mole) $\frac{0.1}{2} \times 0.8 = 0.04 \text{ mole}$

$\therefore$ So, Z = 0.04 × 70 = 2.8 gm $\Rightarrow$ 2.80 gm

$$S \xrightarrow{80\%} U$$

(0.1 mole) $0.1 \times 0.8 = 0.08 \text{ mole}$

$\therefore$ $y = 0.08 \times 68 = 5.44 \text{ gm}$ So, $y + z = 5.44 + 2.80 = 8.24$

26.(4.5) V/T = constant (K)

log V = log T + log K, where $K = \frac{nR}{P}$

For 2 graph; $\log \frac{nR}{P} = 0 = \log 1$

$\frac{nR}{P} = 1 \; (R = P)$; $n = 1 \text{ mol}$; $x_2 = 1 \text{ mol}$

For 1 graph; $\log \frac{nR}{P} = \log 3$; $n = 3$

$x_1 = 3 \text{ mol}$; log 3 graph

$\log \frac{nR}{P} = -\log 2 = \log \frac{1}{2}$; $n = \frac{1}{2} \text{ mol}$

$x_3 = 1/2 \text{ mol}$ $\Rightarrow$ $x_1 + x_2 + x_3 = 4.5 \text{ mol}$

27.(6) Molar ratio after n steps. $\frac{n_{H_2}}{n_{D_2}} = \frac{P^\circ_{H_2}}{P^\circ_{D_2}} \left[\sqrt{\frac{M^\circ_{D_2}}{M^\circ_{H_2}}}\right]^n$

$$\frac{8}{1}=\frac{P}{P}\left[\sqrt{\frac{4}{2}}\right]^n \quad ; \qquad 8=(2)^{n/2} \qquad\qquad 64=2^n$$

$\Rightarrow$ n = 6, 6 steps.

28.(2) $\Delta T_b = iK_b m$

$\Delta T_b = 101.4 - 101 = 0.4$

$0.4 = iK_b m$

For NaCl $i = 2$

$$0.4 = 2 \times K_b \times \frac{0.1}{1}$$

$$K_b = 2K \text{ kg mol}^{-1}$$

29.(16) $E_n = -18 \times \frac{Z^2}{n^2}$ eV/atom

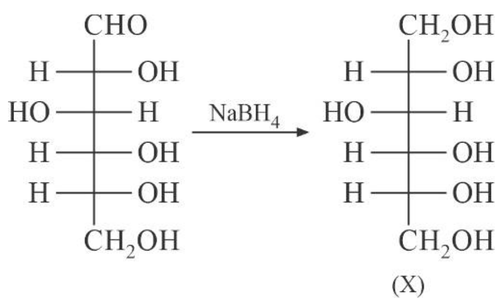
$Z = 1$

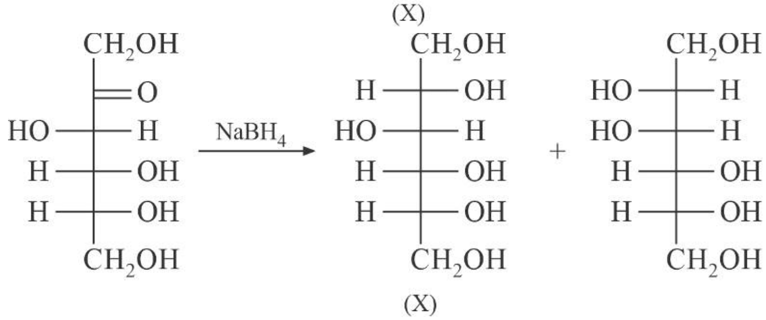
$$\Delta E = E_3 - E_1 = -18 \times 1^2 \times \frac{1}{3^2} + 18 \times 1^2 \times \frac{1}{1^2}$$

$$= 18\left[1-\frac{1}{9}\right] = 18 \times \frac{8}{9} = 16 \text{ eV/atom}$$

30.(AD)

Both D-glucose and D-fructose forms same osazone with $PhNHNH_2$. Optical nature can't be judged by configuration of the chiral centre. D-glucose forms aldonic acid with $Br_2 - H_2O$ but D-fructose doesn't.





31.(ABD)

$$CHBr_3 + OH^- \longrightarrow :CBr_2 + H_2O + Br^-$$

α-elimination

$:CBr_2$ act as electrophile, attack at *ortho* position of phenol to give finally salicyldehyde.

32.(ABCD) Greater the valence of coagulating ion, the greater is the power of coagulation.

Lyophilic sols have greater affinity for dispersion medium, sol particles are heavily solvated, hence greater viscosity.

33.(ABC)

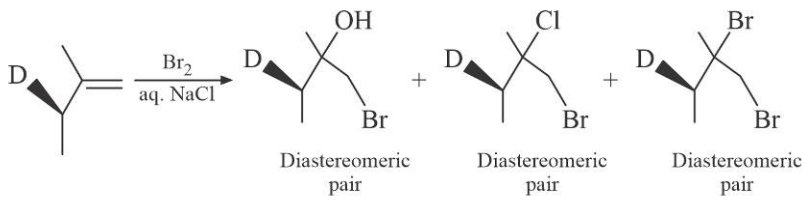
Diagrams of option A, B and C are correct as the PV diagram given in the question. The P–S diagram is not correct.

34.(BCD)

In metallurgy of iron from heametite, Reduction occurs at different range of temperature.

35.(ABC) Brown ring test can be used for analysis NO_3^- of ions.

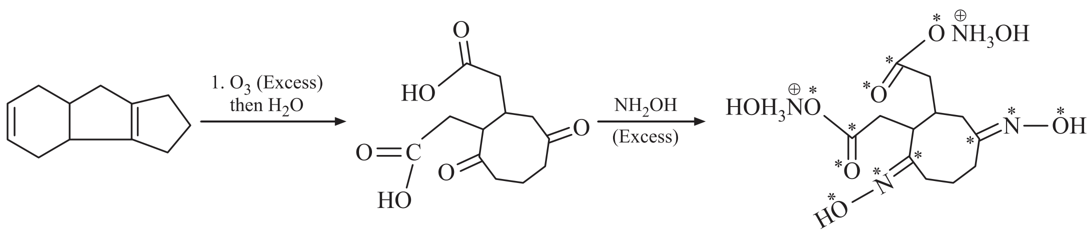
36.(6)



Boiling points of diastereomers are different

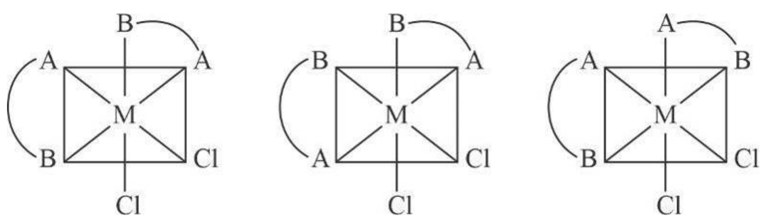
Hence, 3 Diastereomeric pairs are formed and hence 6 fractions will be obtained after fractional distillation.

37.(12)



38.(12) Cis isomers of both the complex are optically active

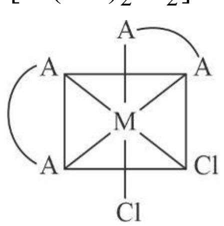
$[M(AB)_2Cl_2]$



(all the above are optically active)

$3 \times 2 = 6$ Isomers (x)

$[M(AA)_2Cl_2]$

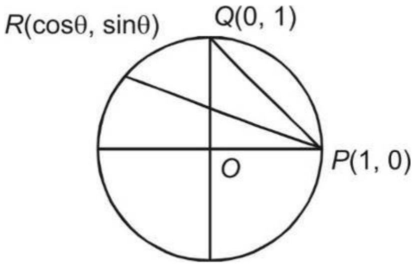


Optically active = 2 isomers (y)

So, $xy = 6 \times 2 = 12$

MATHEMATICS

39.(B)



Let (α, β) be the orthocentre of ΔPQR, then from Euler's line

$\alpha = 1 + \cos\theta, \beta = 1 + \sin\theta$

Locus of (α, β) is $(x-1)^2 + (y-1)^2 = 1 \quad \Rightarrow \quad x^2 + y^2 - 2x - 2y + 1 = 0$

40.(B) $2\int_0^{\pi/2}\left[\left(\cos x - \cos^3 x\right) - \left(x^2 - \frac{\pi^2}{4}\right)\right]dx \quad = 2\int_0^{\pi/2}\left(\sin^2 x.\cos x - x^2 + \frac{\pi^2}{4}\right)dx$

$$= 2\left(\frac{\sin^3 x}{3} - \frac{x^3}{3} + \frac{\pi^2}{4}x\Bigg|_0^{\pi/2}\right) = 2\left(\frac{1}{3} - \frac{\pi^3}{24} + \frac{\pi^3}{8}\right) = \frac{2}{3} + \frac{\pi^3}{6}$$

Here, $a = 6$

$xy = 6$ has 8 integral solutions, which are (2, 3), (3, 2), (1, 6), (6, 1), (–2, –3), (–3, –2), (–1, –6) and (–6, –1).

41.(D) $P(n) = kn^2$

Given $P(1) = K\,; P(2) = 2^2K\,; P(3) = 3^2K\,; P(4) = 4^2K\,; P(5) = 5^2K\,; P(6) = 6^2K$

$\therefore$ Total = 91 K = 1

$\Rightarrow \quad K = \frac{1}{91} \quad \therefore \quad P(1) = \frac{1}{91}\,; P(2) = \frac{4}{91}$ and so on

Let three events A, B, C are defined as

$A : a < b$

$B : a = b$

$C : a > b$

By symmetry,

$P(A) = P(C)$

Also, $P(A) + P(B) + P(C) = 1 \quad \ldots$ (i)

Since, $P(B) = \sum_{i=1}^{6}[P(i)]^2 = \left[\frac{1+16+81+256+625+1296}{91\times 91}\right] = \frac{2275}{91\cdot 91} = \frac{25}{91}$

$2P(A) + P(B) = 1$ [From equation (i)]

$P(A) = \frac{1}{2}[1 - P(B)] = \frac{33}{91} = \frac{p}{q} \Rightarrow \quad p + q = 33 + 91 = 124$

42.(D) $z^{15}=1$

$1+z^8+z^9+z^{10}+....+z^{14} \qquad = 1+\frac{1}{z^7}+\frac{1}{z^6}+....+\frac{1}{z}$

$= \frac{z^7+z^6+z^5+....+z^1+1}{z^7}$

So $\arg\left(\frac{1+z+z^2+....+z^7}{1+z^8+z^9+....+z^{14}}\right) = \arg(z^7) = 7\arg(z) = \frac{14\pi}{15}$

OR

$z^{15}=1$

$\arg\left(\frac{1+z+z^2+z^3+\ldots+z^7}{1+z^{14}+z^{13}+\ldots+z^9+z^8}\right) = \arg\left(\frac{1+z+z^2+\ldots+z^7}{z^{15}+z^{14}+z^{13}+\ldots+z^8}\right) = \arg\left(\frac{1}{z^8}\right) = \arg(z^7) = \frac{14\pi}{15}$

43.(47) $\frac{3^6-3C_1 2^6+3}{3^6} = \frac{20}{27}$

44.(181) $\frac{3C_2(2^6-2)}{3^6} = \frac{62}{243}$

Solution for Q. 45 – 46

45.(1)

46.(125)

$4a^2f(-1)+4af(1)+f(2)=3a^2+3a$

$\underline{4b^2f(-1)+4bf(1)+f(2)=3b^2+3b}$

$4f(-1)(a^2-b^2)+4f(1)(a-b) \qquad = 3(a^2-b^2)+3(a-b)$

$4f(-1)(a+b)+4f(1)=3(a+b)+3$

$4f(-1)=3 \qquad 4f(1)=3$

$f(-1)=\frac{3}{4}, \qquad f(1)=\frac{3}{4},$

$f(2)=0 \qquad 4a+2b+c=0$

$f(2)=0 \qquad a-b+c=\frac{3}{4}$

$\underline{a+b+c=\frac{3}{4}}$

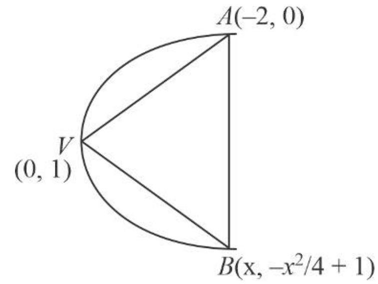
$2b=0 \Rightarrow b=0$

$\underline{4a+c=0}$

$3a=-\frac{3}{4} \Rightarrow a=-\frac{1}{4}; \quad c=\frac{3}{4}+\frac{1}{4}=1$

$f(x)=-\frac{1}{4}x^2+1; \qquad V(\alpha,\beta)=(0,1)$

$A=(-2,0) \Rightarrow P=-2$



$$\frac{-\frac{x^2}{4}}{x}\times\frac{1}{2}=-1\Rightarrow x=8 \qquad B(8,-15)$$

$$y-0=-\frac{3}{2}(x+2)$$

$\Rightarrow \quad 2y=-3x-6 \qquad \Rightarrow \qquad y=-\frac{3}{2}x-3$

$\Rightarrow \quad 3x+2y+6=0$

$$y_1-y_2=\frac{x^2}{4}+1+\frac{3}{2}x+3=-\frac{x^2}{4}+\frac{3x}{2}+4$$

$$\Delta=\frac{\left(\frac{9}{4}-4\times-\frac{1}{4}\times 4\right)^{3/2}}{6\times\frac{1}{16}}=\frac{125}{8}\times\frac{8}{3}$$

OR

a,b,c are the roots of $4x^2 f(-1)+4x\,f(1)+f(2)=3x^2+3x$

or $\quad x^2(4f(-1)-3)+x(4f(1)-3)+f(2)=0$

$\Rightarrow \quad f(-1)=f(1)=\frac{3}{4}$ and $f(2)=0$

$\Rightarrow \quad a=-\frac{1}{4}$ and $c=1$; $\qquad f(x)=-\frac{1}{4}x^2+1$

47.(162)

Let equation of straight line be $\frac{x}{\cos\theta}=\frac{y}{\sin\theta}=r$

Let $\quad OA=r_1, OB=r_2$ and $OC=r_3$

Then $\quad A(r_1\cos\theta, r_1\sin\theta), B=(r_2\cos\theta, r_2\sin\theta) \qquad$ and $\qquad C=(r_3\cos\theta, r_3\sin\theta)$

$\because \quad A$ lies on $x=2 \qquad \Rightarrow \qquad r_1\cos\theta=2$

$\Rightarrow \quad r_1=\frac{2}{\cos\theta} \qquad \ldots$ (i)

$\because \quad B$ lies only $y=3\Rightarrow \qquad r_2\sin\theta=3$

$\Rightarrow \quad r_2=\frac{3}{\sin\theta} \qquad \ldots$ (ii)

and $\quad C$ lies on $x+y=8 \qquad \therefore \qquad r_3\cos\theta+r_3\sin\theta=8$

$$r_3=\frac{8}{\cos\theta+\sin\theta} \qquad \ldots \text{(iii)}$$

$\because \quad r_1\,r_2\,r_3=48\sqrt{2} \qquad \Rightarrow \qquad \left(\frac{2}{\cos\theta}\right)\left(\frac{3}{\sin\theta}\right)\frac{8}{(\cos\theta+\sin\theta)}=48\sqrt{2}$

$\Rightarrow \quad \sqrt{2}\cos\theta\sin\theta(\cos\theta+\sin\theta)=1 \quad \Rightarrow \quad \sin 2\theta \sin\left(\theta+\frac{\pi}{4}\right)=1$

$\therefore$ Both $\sin 2\theta$ and $\sin\left(\theta+\frac{\pi}{4}\right)$ should be equal to 1

So, $\theta=\frac{\pi}{4}$

$\therefore$ From (i), $r_1=2\sqrt{2}$

From (ii) $r_2=3\sqrt{2}$

From (iii), $r_3=\frac{8}{\sqrt{2}}=4\sqrt{2}$

Hence, $OA+OB+OC=r_1+r_2+r_3=9\sqrt{2}$

48.(1) The equation of line OA is $\frac{x-0}{\cos\frac{\pi}{4}}=\frac{y-0}{\sin\frac{\pi}{4}} \Rightarrow x-y=0$

49.(ACD)

(A) $a_{ij}=-a_{ji} \Rightarrow a_{ij}+a_{ji}=0 \Rightarrow A$ is skew symmetric matrix

(C) $A^2=2A \Rightarrow A^3=2^2A \Rightarrow A^6=2^5A$

(D) A^6B^7 is a skew symmetric matrix of odd order

50.(ABCD)

(A) $$\frac{x-\frac{1}{x}}{4\left(x^2+\frac{1}{x^2}-1\right)}=\frac{x-\frac{1}{x}}{4\left(\left(x-\frac{1}{x}\right)^2+1\right)}$$

So, $-\frac{1}{8}\le \frac{x-\frac{1}{x}}{4\left(\left(x-\frac{1}{x}\right)^2+1\right)}\le\frac{1}{8} \quad \therefore \quad \left[\frac{x(x^2-1)}{4(x^4-x^2+1)}+\frac{1}{8}\right]=0$

So, $f(x)=2x+[x]+\frac{1}{2}\sin 2x$ injective but not onto

(B) Many one $f(0)=f(-2)=0$

(C) f is not one – one

(D) Obviously f is even

51.(AB)

Probability will be independent of balls being identical or distinct. The required probability is same as the probability that the first ball drawn is black which is equal to $\frac{m}{m+n}$.

52.(ABD)

$(2P+I)(4P^2-2P+I)=8P^3+I^2=I$

Similarly, $(I-2P)(I+2P+4P^2)=I$, if P has integer entries, then $|I-2P|$ and $|I+2P|$ will be equal to either 1 or –1

53.(BCD)

$$p=\sec^2(\sec^{-1}\sqrt{5})+\operatorname{cosec}^2(\operatorname{cosec}^{-1}\sqrt{10})$$

$p=15$ and

$$\tan^{-1}\left(\frac{|\sec 8|+1}{\tan 8}\right)=\tan^{-1}\left(\frac{-1+\cos 8}{\sin 8}\right)=\pi-4$$

$\therefore \quad q=\pi-(\pi-4) \quad \Rightarrow \quad \boxed{q=4}$

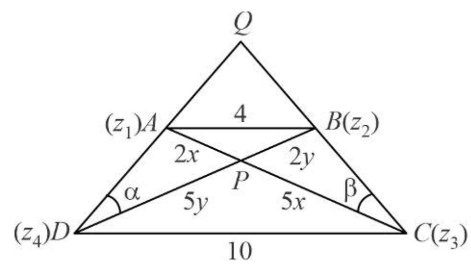
$\therefore \quad pq=15\times 4=60$

$\therefore$ (B), (C) and (D) options are correct

54.(AC)

ΔABP and ΔCDP are similar if $AP=2x$

$BP=2y$ then $CP=5x, DP=5y$



Area of trapezium $ABCD=\frac{49}{2}xy$

$$\tan\alpha=\frac{2x}{5y},\tan\beta=\frac{2y}{5x}$$

Also, $\alpha+\beta=45° \quad \Rightarrow \quad xy=\frac{10}{21}(x^2+y^2)$

Also, $AB^2=AP^2+BP^2 \quad \Rightarrow \quad x^2+y^2=4$

$\Rightarrow \quad xy=\frac{40}{21}; \quad \quad \text{Ar}(\Delta PCD)=5xy$

55.(6) Here $P(x)=Q(x)\ (x^2+1)+x^2-x+1$

$$\therefore \quad \sum P(\alpha)=\sum\alpha^2-\sum\alpha+4=\left(\sum\alpha\right)^2-2\sum\alpha\beta-\sum\alpha+4=1+2-1+4=6$$

56.(3) $x^3-11x^2+36-36=0, r_1, r_2, r_3$ are roots

Required area $=\frac{1}{2}ar_1+\frac{1}{2}br_2+\frac{1}{2}cr_3+\Delta$

$$=\frac{1}{2}\left(as\tan\frac{A}{2}+bs\tan\frac{B}{2}+cs\tan\frac{C}{2}\right)+\Delta$$

$$=\frac{1}{2}s\left(2R\sin A\tan\frac{A}{2}+2R\sin B\tan\frac{B}{2}+2R\sin C\tan\frac{C}{2}\right)+\Delta$$

$$=\frac{2R}{2}s\left(2\left(\sin^2\frac{A}{2}+\sin^2\frac{B}{2}+\sin^2\frac{C}{2}\right)\right)+\Delta$$

$$=2sR\left(\frac{3}{2}-\frac{(\cos A+\cos B+\cos C)}{2}\right)+\Delta=2sR\left(1+\frac{r}{2R}\right)+\Delta$$

$$=2sR-\Delta+\Delta=2sR \qquad \ldots \text{(i)}$$

$$\frac{1}{r_2}+\frac{1}{r_2}+\frac{1}{r_3}=\frac{1}{r}=\frac{36}{36} \qquad \Rightarrow \qquad r=1$$

and $\quad r_1+r_2+r_3=4R+r\,; \quad R=5/2$

and $\quad r_1r_2r_3=\Delta^2 \quad \Rightarrow \quad 36(1)=\Delta^2 \quad \Rightarrow \quad \Delta=6$ and $s=6 \quad \therefore \quad 2sR=30$

57.(16) S.D. between given lines

$$=1=\left|\frac{(\vec{b}-\vec{a})\cdot(\vec{p}\times\vec{q})}{|\vec{p}\times\vec{q}|}\right| \qquad \Rightarrow \qquad 1=\left|\frac{|\vec{b}-\vec{a}|\cdot|\vec{p}\times\vec{q}|\cos 60^\circ}{1/2}\right|$$

$$=\frac{|\overrightarrow{AB}|\cdot\left(\frac{1}{2}\right)\cdot\frac{1}{2}}{(1/2)}$$

$\Rightarrow \qquad 2=AB=k$ (given) = (Area of parallelogram $=|\vec{p}\times\vec{q}|=\frac{1}{2}$ (given))

$\therefore \qquad \lambda^4=2^4=16$